



# 1998-99 CATS ASSESSMENT

## Open-Response Item Scoring Worksheet

### Grade 8—Mathematics

The **academic expectation** addressed by “Martin’s Three-Digit Number” is

2.7 Students understand number concepts and use numbers appropriately and accurately.

The **core content** assessed by this item includes

Number/Computation Concepts

- Students should understand place value of large and small numbers.

Number/Computation Skills

- Students should be able to determine prime numbers, composite numbers, factors, multiples, greatest common factors, least common multiples.

#### Martin’s Three-Digit Number

Martin said, “I am thinking of a whole number between 100 and 300.

- The number is divisible by 3 but not by 9.
- The ones digit is the sum of the hundreds digit and the tens digit.”

- a. Show why 153 cannot be Martin’s number.
- b. Find **all** the numbers that match Martin’s clues. Show all your work.
- c. Write one more clue that would limit the answer in **part b** to one and only one correct number.



## SCORING GUIDE

### Grade 8 Mathematics

Score	Description
4	Student scores 6 points.
3	Student scores 4 to 5.5 points.
2	Student scores 2 to 3.5 points.
1	Student scores .5 to 1.5 points. <b>OR</b> Student shows minimal understanding of divisibility.
0	Response is totally incorrect or irrelevant.
Blank	No response.

#### Correct answers:

**part a:** 153 is divisible by 9, or  
 $1 + 5 = 6$ , or  
 $1 + 5 \neq 3$

**part b:** 123, 156, 213, 246

**part c:** response that would result in only one correct number

**NOTE:** There should be NO incorrect numbers at the 4 point score.  
Need at least 2 numbers in part b to get points for part c.



# SCORING GUIDE

## Grade 8 Mathematics

### Score points:

#### part a:

score 1 point for complete strategy shown

**OR**

score 0.5 points for strategy explained but calculation not shown

#### part b:

score 0.5 points for each correct answer (total: 2 pts.)

score 1 point for divisible by 3 but not by 9 shown (calculations)

**OR**

score 0.5 point for divisible by 3 but not by 9 explained (no calculations)

**OR**

score 0.5 point for divisible by 3 shown but divisible by 9 not shown (or vice versa) (no calculations)

**OR**

score 0.5 point for list of numbers that are divisible by 3 but not by 9 (check 108 and a few others)

score 1 point for sum of digits shown (calculation)

**OR**

score 0.5 point for sum of digits explained (no calculations)

#### part c:

score 1 point for clue that would limit answer in part b to one number

**OR**

score 0.5 point for weak clue (clue that does not require other clues to determine number—

e.g.,  $492 \div 2 = \text{the number}$ )



# ANNOTATED STUDENT RESPONSE

## Grade 8 Mathematics

### Sample 4-Point Response of Student Work

#### Student Response

A. 153 cannot be Martin's number because it is divisible by 9.  $153 \div 9 = 17$ .

B. All the numbers that match Martin's clues are 213, 123, 246, 156.

$213 \div 3 = 71$  divisible by 3  
 $213 \div 9 = 23.666$  not divisible by 9  
 $2 + 1 = 3$  hundreds and tens digit add up to three

$123 \div 3 = 41$  divisible by 3  
 $123 \div 9 = 13.666$  not divisible by 9  
 $1 + 2 = 3$  sum of hundreds and tens digits is 3

$246 \div 3 = 82$  divisible by 3  
 $246 \div 9 = 27.333$  not divisible by 9  
 $2 + 4 = 6$  sum of hundreds and tens digits is 6

$156 \div 3 = 52$  divisible by 3  
 $156 \div 9 = 17.333$  not divisible by 9  
 $1 + 5 = 6$  sum of hundreds and tens digits is 6

C. One more clue that would limit the answer in part b to one and only one correct number would be that the tens digit is 2.

Answer = 123.

Student provides work to show why 153 cannot be Martin's number (i.e.,  $153 \div 9 = 17$ ).

Student finds the correct values for part b.

Student provides calculations to show that the sum of the hundreds digit and the tens digit equals the ones digit for each of the numbers; each number is divisible by 3 but is not divisible by 9.

Student provides calculations to show that the number is divisible by 3 for each of the numbers.

Student provides calculations to show that the number is not divisible by 9 for each of the numbers.

Student gives a correct clue that would limit Martin's numbers to one number (i.e., the tens digit is 2).

Overall, the student demonstrates a solid understanding of number concepts and of divisibility and place value by fully responding to the prompt with complete and correct explanations.



# ANNOTATED STUDENT RESPONSE

## Grade 8 Mathematics

### Sample 4-Point Response of Student Work

#### Student Response

A.) 153 cannot be Martin's number because when you follow all of Martin's clues  $1+5$  is 6 and in order to be Martin's number  $1+5$  would have to equal 3. Another reason 153 is not Martin's number is because 153 is divisible by both 3 and 9.

B.) All of the numbers that match Martin's clues are 123, 156, 213, and 246. These numbers match Martin's clues because they are all between 100 and 300, all of their ones digits are the sums of their hundreds and tens digit, and they are all divisible by 3 and not 9.

C.) One more clue that would limit the answer in part b to one and only one answer would be that the ones, tens, and hundreds digits must all be even numbers. This clue would limit the answer to the number 246.

$1+2 = 3$	41	$13.\overline{6}$
$1+5 = 6$	$3\overline{)123}$	$9\overline{)123}$
$2+1 = 3$	52	$17.\overline{3}$
$2+4 = 6$	$3\overline{)156}$	$9\overline{)156}$
	71	$23.\overline{6}$
	$3\overline{)213}$	$9\overline{)213}$
	82	$27.\overline{3}$
	$3\overline{)246}$	$9\overline{)246}$

Student provides work to show why 153 cannot be Martin's number (i.e.,  $1 + 5$  is 6, 153 is divisible by both 3 and 9).

Student finds the correct values for part b and explains why they are the numbers that match Martin's clues.

Student provides calculations to show that the sum of the hundreds digit and the tens digit equals the ones digit; each number is divisible by 3 but is not divisible by 9.

Student gives a correct clue that would limit Martin's numbers to one number (i.e., the ones, tens and hundreds digits must all be even numbers).

Overall, the student demonstrates a solid understanding of number concepts and of divisibility and place value by fully responding to the prompt with complete and correct explanations.



# ANNOTATED STUDENT RESPONSE

## Grade 8 Mathematics

### Sample 3-Point Response of Student Work

#### Student Response

A.  $1 + 5 = 3$  153 cannot be the answer because the sum of the hundreds and tens digits is not equal to the ones digit.

B. 123  
 $1 + 2 = 3$

$$\begin{array}{r} 41 \\ 3 \overline{)123} \\ \underline{12} \phantom{0} \\ 03 \\ \underline{3} \\ 0 \end{array}$$

156  
 $1 + 5 = 6$

$$\begin{array}{r} 52 \\ 3 \overline{)156} \\ \underline{15} \phantom{0} \\ 06 \\ \underline{6} \\ 0 \end{array}$$

213  
 $2 + 3 = 3$

$$\begin{array}{r} 71 \\ 3 \overline{)213} \\ \underline{21} \phantom{0} \\ 03 \\ \underline{3} \\ 0 \end{array}$$

246  
 $2 + 4 = 6$

$$\begin{array}{r} 82 \\ 3 \overline{)246} \\ \underline{24} \phantom{0} \\ 06 \\ \underline{6} \\ 0 \end{array}$$

C. All the digits are even numbers.

Student provides work to show why 153 cannot be Martin's number (i.e.,  $1+5 = 3$ , not 6).

Student finds the correct values for part b.

Student provides calculations for each number to show that the sum of the hundreds digit and the tens digit is the ones digit and the number is divisible by 3. However, the student does not show that the number is not divisible by 9.

Student gives a correct clue that would limit Martin's numbers to one number (i.e., all the digits are even).

Overall, the student demonstrates a general understanding of divisibility and place value by correctly answering all three parts and showing most of the work.



# ANNOTATED STUDENT RESPONSE

## Grade 8 Mathematics

### Sample 2-Point Response of Student Work

#### Student Response

- A. 153 cannot be Martian's number because it is divisible by 9.

$$\begin{array}{r} 17 \text{ R } 0 \\ 9 \overline{)153} \\ \underline{153} \\ 0 \end{array}$$

- B. All the numbers that match Matian's clues are: 123, 213, 156, 246

- C. Every third number cross if off for ex: 102, 105, ~~106~~, , , 300. Then every number with a three or a six on the end that hasn't been marked off is your 4 to choose from.

Student provides work to show why 153 cannot be Martin's number (i.e.,  $153 \div 9 = 17$ ).

Student provides correct values for part b; however, student does not provide any work.

Student provides an answer for part c. However, rather than providing a clue that would limit answers from part b to one number, student provides a limited explanation of how answers to part b were determined.

Overall, the student demonstrates some understanding of divisibility and place value by correctly answering two parts and showing work for one part.



# ANNOTATED STUDENT RESPONSE

## Grade 8 Mathematics

### Sample 1-Point Response of Student Work

#### Student Response

$$\begin{array}{r} 17 \\ 9 \overline{)153} \\ \underline{9} \phantom{00} \\ 63 \phantom{00} \\ \underline{63} \phantom{00} \\ 0 \phantom{00} \end{array} \quad \begin{array}{r} 53.\overline{3} \\ 3 \overline{)160.0} \\ \underline{15} \phantom{00} \\ 10 \phantom{00} \end{array} \quad \begin{array}{r} 153 \\ 9 \overline{)153} \\ \underline{9} \phantom{00} \\ 12 \phantom{00} \\ \underline{9} \phantom{00} \\ 30 \phantom{00} \end{array} \quad \begin{array}{r} 156 \\ 9 \overline{)159} \\ \underline{9} \phantom{00} \\ 69 \phantom{00} \end{array} \quad \begin{array}{r} \overline{)162} \end{array}$$

It is the lowest number that can be divide by 3 but not by 9.

← Student provides work to show why 153 cannot be Martin's number (i.e.,  $153 \div 9 = 17$ ).

← Student attempts to find other numbers that match Martin's clues; however, work is unclear and incomplete. Explanation indicates that the student has incorrectly interpreted the clues (i.e., "it is the lowest number that can be divided by 3 but not by 9").

Overall, the student shows a limited understanding of divisibility.



# INSTRUCTIONAL STRATEGIES

## Grade 8 Mathematics

The open-response item “**Martin’s Three-Digit Number**” is designed to assess students’ understanding of number concepts. More specifically the item assesses students’ understanding of place value and divisibility (i.e., a number divisible by 3 but not by 9) and their use in determining the number that matches the given clues. The instructional strategies below present ideas for helping students explore and master these concepts.

Review place value.

Review vocabulary (i.e., digit, sum, between, divisible).

Review divisibility of numbers.

Provide students with multiple opportunities to work on a variety of math problems that require them to decipher and organize information (e.g., CATS-like open-response items). Teach students a variety of strategies for organizing information (e.g., using tables, charts, graphs, highlighting and underlining strategies) as they solve problems and make comparisons or show relationships. Such opportunities can help students learn to use information to write equations, check their reasoning and the reasonableness of their answers, document their thinking, and explain their work to others.

Provide opportunities for students to work individually, in pairs, in small groups, and/or as a class to complete (with teacher guidance and support) any or all of the following activities:

- Practice using one or more tests of divisibility on a number.
- Practice using clues to narrow a set of numbers down to one or more numbers.
- Create and use a variety of clues to determine a single number from a set of numbers.
- Discuss and write about strategies for solving number puzzles from mathematical clues. This can help students develop and/or refine their ability to effectively communicate their mathematical thinking both verbally and in writing. Prior to these activities, the teacher can model strategies for communicating mathematical thinking.